On *Matter and Motion*

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Maxwell's little book *Matter and Motion* is of the sort that is easily overlooked. It is brief, modest, and admittedly "elementary"; at first scanning, it seems not to get very far with the subject of mechanics. There are few equations, it is not replete with illustrations or examples, and there are no worked problems. It is also well over a century out of date. In sum, then, it seems at best the kind of introductory text one might recommend to a beginner whose ultimate ambitions lay in other directions.

The error entailed in such a judgment has to do, I suspect, with our understanding of the term *elementary*. In ordinary parlance the word suggests the superficial, the simplified—something watered down for beginners. Though Maxwell certainly is aiming his book at beginners, I think he has the opposite notion of what is elementary, or what it might mean to place ourselves at the threshold of a science. The elements of a science lie not on its surface but in its depths. They are few, and in a sense uncomplicated, and in that respect they are appropriate for beginners: indeed, beginners may be in a better position to appreciate them than experts, whose long experience with complexities often makes the simplest things invisible.

Maxwell is evidently concerned, himself, with the foundations of this science of mechanics, and he seems to enjoy taking his case for their reformulation, not to his fellow scientists, but to the workingman. Maxwell taught over the years in classes of adult education, and this little treatise, as the title page tells us, was published by the Committee of General Literature and Education of the Society for Promoting Christian Knowledge. As I discuss elsewhere in this issue of *The Great Ideas Today*, Maxwell was greatly drawn to the figure of Michael Faraday, a brilliantly productive scientist who was innocent of any knowledge of mathematics beyond the operations of arithmetic. It seems important to Maxwell to be able to express the outcome of sophisticated mathematical investigations in plain prose, "forms of words," as he says, which can be understood by the unmathematical reader.

At the time *Matter and Motion* was published, Maxwell was heavily engaged in a new enterprise at Cambridge University. He had recently been appointed the university's first professor of experimental physics; he was launching the new Cavendish Laboratory, and in connection with this assignment he was giving lectures on mechanics, electricity and magnetism, and heat to students who were not skilled in mathematics. This was a sharp turn for him from his own background, for as a student at Cambridge he had taken honors in the "mathematical" examinations, which in fact covered much of the mathematical physics of the day, and he had only recently published his *Treatise on Electricity and Magnetism* and a text for Cambridge students who were studying that subject within the context of a severe discipline in analytic mathematics. In his lectures at the Cavendish, then, Maxwell was regularly translating his mathematical ideas into new nonmathematical formulations, using models, illustrative experiments, and careful prose. Out of this was coming his *Elementary Treatise in Electricity and Magnetism*, which very unfortunately he did not complete before his death. I presume the present *Matter and Motion* represents the kind of foundation he was laying in mechanics for his nonmathematical Cambridge students. Maxwell makes it clear that he himself prefers this elementary mode of presentation; he does not think of it as intellectually inferior or inappropriate to the subject.

It is evident that Maxwell approaches the foundations of mechanics with the spirit of a reformer. Not only does he set out to purge certain important errors, but even in a science as old and established as mechanics, he sees a new kind of possibility which will come with a reformation of the elements. The beginning students, then, immediately find themselves caught up in a workshop in which the most fundamental of the physical sciences is undergoing reconstruction. The reader of *Matter and Motion* is in a specially privileged position, joining with Maxwell in reshaping the first of the sciences.

Certainly *Matter and Motion* is unlike other elementary texts, of Maxwell's time or ours. The reader is in a sense given no quarter but taken straight to the heart of the difficulty. There is no preliminary *terra firma* in Maxwell's book: we don't begin with "simple" cases, in which a stone stands firmly on the ground and a lever is applied to it, or a ball is dropped from a given height at a given time, to fall a specified distance. Instead, from the outset we find ourselves dealing with systems of bodies, whose positions are defined, not with respect to a fixed earth, but in relation to one another. Such a system, we are told, may be small or it may be the whole universe. The term *configuration* is introduced immediately thereafter, very abstractly, as "the assemblage of relative positions."

Two themes are thus introduced in full earnestness from the beginning: that of the wholeness of a *system* of bodies, the whole to be treated as an entity prior to the separate consideration of its parts; and that of the relativity of all of our accounts of physical systems. Without arguing that *Matter and Motion* itself had any particular influence, we can see that it stands at a historical turning point in physics, and that it very consciously is transforming the foundations of mechanics. The new concept which has made it possible to take the system as the object of scientific reasoning is that of *energy*, with the principle of its conservation. The principle of relativity, which in *Matter and Motion* goes hand in hand with that of the wholeness of the system, Maxwell probably takes more directly from metaphysical convictions.

It is important to Maxwell to make clear that this is a treatise not about physics but about abstract dynamics, that is, a science of axiomatic principles and their consequences, to be contrasted with physics, which takes observed phenomena into consideration. Throughout *Matter and Motion* Maxwell dwells on consideration of our most fundamental ideas: ideas of time, of space, of matter, force, and energy. They are the subject matter of this treatise, and one of its major concerns is to reshape, clarify, and purify these ideas and the science based on them. Thus, it is not at all inappropriate that in Article 15 he explicitly turns to "a few points relating to the metaphysics" of the subject of space. In so doing, he joins issue with Descartes in a way which might suggest not only his disagreement but a deeper affinity of purpose. Descartes, Maxwell goes on to say in Article 16, was very wrong about the definition of matter, not having understood its identification with the principle of inertia, and hence was in turn wrong about the idea of space. Matter, Maxwell insists, is one thing, space, another—Descartes had in effect identified them.

With this fundamental correction, however, I think Maxwell comes close to pursuing the Cartesian goal of a study of mechanics from the point of view of thought alone: in abstract dynamics we are dealing with pure concepts. Thus the Cartesian title, alluding to the two principles of the Cartesian account of the physical world, is telling us where Maxwell stands. Whereas Newtonian forces must be expressed between specific bodies, it is possible to work with the energy of an entire system as a whole. Each configuration of a conservative system will correspond to a definite energy, and from state to state of such a system we will pass through definite energy changes. In these terms, dynamics can mount to the higher ground of the "configuration" and characterize systems of wholes and their relationships as its primary concern. Thus Maxwell points out that where Newton spoke of a "force" of one body on another, the larger view is that of "stress," which sees the relativity of the two bodies in their mutuality.

All this takes us, however, directly to what is probably the central thesis of *Matter and Motion*, surely a powerful insight into the central problem of dynamics as well as the future course of physics: the principle of relativity. Maxwell is unyielding on this concept, which he sees as belonging to our ideas themselves, not as contingent upon the phenomena of the world. Our common words seem to speak of absolutes, and to make a distinction between, for example, "relative" and "absolute" space. But for Maxwell it is a fundamental, metaphysical truth that "All our knowledge, both of time and place, is essentially relative" (art. 18).

Maxwell must feel, then, that Newton was on the wrong track in "conceiving" absolute time and space as the foundations of the mathematical theory of mechanics, and then recognizing by contrast that phenomenal time and space are relative. Rather, the foundations of dynamics must speak of these concepts as *inherently* relativistic. Einstein, not Maxwell, carries this through to completion for both space and time; but Maxwell here certainly defines the task.

One way to present such a relativistic view of a system to the mind of the reader is by way of a diagram, and for Maxwell the notion of a diagram as a representation of a physical system is very important. As scientific editor of the Ninth Edition of the *Encyclopædia Britannica*, he introduced an article, which he wrote himself, on "Diagrams"—a topic that would not have occurred to most editors. There are many instances of diagrams in *Matter and Motion*, because they express both our knowledge and its limitations in a single image. Maxwell gives us diagrams, not only of the positions of systems of bodies, but of displacements, velocities, and accelerations. The important thing about his diagrams is that they have no origins—that is, they do not locate their points with respect to a supposed coordinate system: there is no frame of reference. Points are located and move and accelerate only with respect of one another:

> This diagram of displacements (without an origin) will then represent neither more nor less than all we can ever know about the displacement of the system (art. 22).

If Maxwell is to remain true to the principle of relativity as he states it, the difficulty is not that we are doomed to ignorance, but that position and displacement are *inherently* relative: there is, in truth, no origin.

In Article 29 of *Matter and Motion*, Maxwell presents the diagrams of position and velocity in a way that dramatizes, I believe, his insight concerning relativity. The diagram of position is rightly labeled "Diagram of Configuration," since in a sense *position* would suggest absolute location, whereas the term *con-figuration* expresses immediately the fact that the only "figure" that emerges is a pattern of points relative to one another. Maxwell has presented this diagram of configuration, not only without an origin, but printed as bright dots on a black field, so that not even the paper appears to us as *terra firma*; the bodies which make up a system appear naked, like points of light in the night sky. Between them there are no absolute measures of distance any more than there are in the cosmos.

The transformed dynamics that emerges from the pages of *Matter and Motion* is not a mere structure of thought but a working instrument at the cutting edge of science—these dynamic ideas are, Maxwell says, "scientific," i.e., science-*making*. The reader in effect is invited to stand with

Maxwell at the borders of the unknown, and to survey with him the terrain which lies beyond. Ultimately, Maxwell perceives that a completed dynamics would be coextensive with the whole of physical science, projected into "dynamical form."

Maxwell refers the reader to the efforts he is currently making to accomplish this dynamical theory in the realm of electromagnetism, in his *Treatise on Electricity and Magnetism* (1873). The dynamical approach brings to each physical problem only the conviction that it can be described in the abstract terms of dynamics; it is thus the minimal, and hence safest, hypothesis. Maxwell sees, as well, in Article 148, its promise for molecular physics.

One problem, however, constitutes the burden of the latter part of the book: the problem of the aether. Maxwell says, "Energy cannot exist except in connection with matter." But energy, in the form of light and heat, traverses the space between the sun and the earth. Hence, there must be matter "existing in the interplanetary spaces" and, in fact, "disseminated through the whole of the visible universe (art. 108). This "matter," the "aether," cannot be gravitational in proportion to its mass. Yet Newton, by means of experiments reported in the *Principia*, demonstrated that all matter exerts a gravitational force corresponding to its mass. Beginning with Chapter VII, Maxwell therefore very carefully reviews the theory of the pendulum and its application to the measurement of the gravitational constant. In Chapter VIII he concludes, notwithstanding the experimental evidence, that "it is still extremely doubtful whether the medium of light and electricity is a gravitating substance, though it is certainly material and has mass" (art. 145). The implicit question, standing as it does simply as an enigma, in effect sets the stage for much of the physics of the twentieth century.

In reflecting on the relation between *Matter and Motion* and the physics of the twentieth century, it is important to recall the breadth of Maxwell's claim for the inherent relativity of the concepts of time and space. It is sometimes supposed that Maxwell, in searching for that matter which conveyed the energies of light and electricity, hoped to find an absolute reference frame and thereby escape the implications of relativity. Maxwell makes clear in this text, I believe, that his sense of relativity cuts much deeper than that:

> To discover the existence of a medium, and to determine our velocity with respect to it by observation on the motion of bodies, is a legitimate scientific inquiry, but supposing all this done we should have discovered, not an error in the laws of motion, but a new fact in science (art. 48).

Motion relative to an aether would then be an important physical fact, but it would not alter the foundations of dynamics. Those are relativistic in their very conception. As I conclude in an article elsewhere in this volume, Maxwell would, then, remain a relativist even in the face of the discovery of a universal aether. The insights of this little "elementary," treatise cut very deep.